Magnetic-field dependence of electron spin relaxation in n-type semiconductors

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We present a theoretical investigation of the magnetic field dependence of the longitudinal (T_1) and transverse (T_2) spin relaxation times of conduction band electrons in n-type III-V semiconductors. In particular, we find that the interplay between the Dyakonov-Perel process and an additional spin relaxation channel, which originates from the electron wave vector dependence of the electron g-factor, yields a maximal T_2 at a finite magnetic field. We compare our results with existing experimental data on n-type GaAs and make specific additional predictions for the magnetic field dependence of electron spin lifetimes.

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The electron spin in a semiconductor is a robust object which can be utilized to add new functionality to existing electronic devices or to even build completely new devices based on this spin degree of freedom. [1] Establishment of successful spintronics devices requires a thorough understanding of the electron spin dynamics in a semiconducting environment. In particular, spin relaxation processes need to be identified and controlled.

Important electron spin relaxation processes in n-type semiconductors include the Elliott-Yafet (EY) process [2, 3], that leads to spin flip scattering and, in semiconductors without inversion symmetry, the Dyakonov-Perel (DP) process [4], in which spin states precess because of spin off-diagonal Hamiltonian matrix elements resulting from a combination of the spin-orbit interaction and inversion asymmetry. Typically, the DP mechanism dominates the spin dynamics in n-type III-V semiconductors. An external magnetic field, in many cases required for control and manipulation in spintronics devices, can significantly influence electron spin dynamics. A magnetic field has two main effects on electron spin relaxation: (i) it quenches the DP process thereby tending to extend the spin lifetimes as a function of the magnetic field [5], and (ii) it opens an additional spin relaxation process which tends to reduce the spin lifetimes in applied magnetic fields. [6] The latter process is due to the wave vector dependence of the conduction band (CB) electron g-factor. As a result of the variations in the g-factor, electrons in different quantum states precess about a transverse magnetic field at different rates and thus lose spin coherence. For brevity we will refer to this process as a variable g-factor (VG) mechanism.

In contrast to previous studies [2–11] of spin relaxation in (bulk) n-type III-V semiconductors, we simultaneously treat the EY, DP, and VG processes on an equal footing and focus on the interplay between the various spin relaxation processes as a function of the magnetic field. Thereby, we are able to study in detail the competition between the quenching of the DP process and the appearance of the VG process.

Specifically, we calculate the longitudinal (T_1) and transverse (T_2) spin relaxation times as a function of temperature, electron density and magnetic field. We find that the VG process dominantly influences the transverse (T_2) spin relaxation time. In particular, as a result of the competition between the quenching of the DP process and the introduction of the VG process, there is a magnetic field for which the transverse (T_2) spin lifetime is maximal. From the slope of T_2 at small magnetic fields it is moreover possible to determine whether the DP or the EY process dominates spin relaxation at zero magnetic field. In contrast, the magnetic field dependence of the longitudinal (T_1) lifetime is essentially unaffected by the VG process and dominated by the quenching of the DP process. Thus, they generally increase with field and saturate at a value given by the EY process.

In an applied magnetic field, the CB electrons in a III-V semiconductor, e.g. GaAs, are described by the Hamiltonian [12, 13]

$$\mathcal{H}_{\alpha\beta}(\vec{K}) = \epsilon(\vec{K})\delta_{\alpha\beta} + \frac{\hbar}{2}[\vec{\Omega}_L + \vec{\Omega}_{IA}(\vec{K}) + \vec{\Omega}_g(\vec{K})]\cdot\vec{\sigma}_{\alpha\beta}, \quad (1)$$

where $\vec{K} = \vec{k} - (e/\hbar c)\vec{A}(\vec{r})$ [$\vec{A}(\vec{r})$ is the vector potential], $\epsilon(\vec{k})$ is the Kramers degenerate dispersion of CB electrons, $\hbar\vec{\Omega}_L = \mu_B g^* \vec{B}$ is the Larmor frequency, $\hbar\vec{\Omega}_{IA}(\vec{K}) = 2\delta_0\vec{\kappa}(\vec{K})$ is the splitting of the CB dispersion due to the combination of spin-orbit interaction and inversion asymmetry, and

$$\hbar\vec{\Omega}_g(\vec{K}) = 2a_4 K^2\vec{B} + 2a_5\{\vec{K}, \vec{B}\cdot\vec{K}\} + 2a_6\vec{\tau}(\vec{K}, \vec{B}) \quad (2)$$

is a term which gives rise to a wave vector dependence of the CB electron g-factor. The definitions of the vectors $\vec{\kappa}(\vec{K})$ and $\vec{\tau}(\vec{K}, \vec{B})$ and of the parameters δ_0, $a_i, i = 4, 5, 6$ are given in Refs. [12, 13] and $\{,\}$ indicates an anticommutator.

Our calculation starts from the full quantum kinetic equations for the contour-ordered Green functions [14], from which we derive, considering a classical homogeneous magnetic field and using the fact that wave vector scattering is essentially instantaneous on the time scale of spin relaxation, a semiclassical kinetic equation for the CB electron density matrix. We then linearize this kinetic equation with respect to the CB electron spin density, assuming, as an initial condition, small spin polarization. Treating scattering processes in the Born approximation and expanding the collision integrals up to second order in the wave vector transfer (diffusion approximation [14]), we finally obtain a generalized Fokker-Planck-Landau equation for the spin density $\vec{S}(\vec{k}t)$ which, in atomic units (with magnetic field along the z-axis) reads

$$[\partial_t - i\Omega_C \hat{\mathcal{L}}_z\ ]\vec{S}(\vec{k}t) = [\vec{\Omega}_L + \vec{\Omega}_{IA}(\vec{k}) + \vec{\Omega}_g(\vec{k})] \times \vec{S}(\vec{k}t) + [\hat{\mathcal{D}} - \frac{1}{2\tau_1(k)}\hat{\mathcal{L}}^2]\vec{S}(\vec{k}t) - \Gamma(\vec{k})\vec{S}(\vec{k}t), \tag{3}$$

with Ω_C the cyclotron frequency, $\hat{\mathcal{L}}_z$ and $\hat{\mathcal{L}}^2$ the z-component and the squared total angular momentum operator in wave vector space, respectively, $1/\tau_1(k)$ the sum of the (on-shell) wave vector relaxation rates for the various scattering processes, and $\hat{\mathcal{D}}$ a differential operator in $k = |\vec{k}|$ relevant to inelastic scattering processes.

Equation (3) contains EY, DP, and VG processes and accounts for Larmor precession and orbital motion of the CB electrons in the magnetic field. More specifically, the EY process, due to genuine spin flip scattering events, is given by the tensor Γ, whereas the DP and VG processes originate from the interplay of spin conserving wave vector scattering events described by the differential operator $\hat{\mathcal{D}} - (1/2\tau_1)\hat{\mathcal{L}}^2$ and the torque forces due to $\vec{\Omega}_{IA}$ and $\vec{\Omega}_g$, respectively. The orbital motion encoded in $-i\Omega_C\hat{\mathcal{L}}_z$ and, to a lesser extend, the torque force due to $\vec{\Omega}_L$ lead to a quenching of the DP process.

It is possible to derive from Eq. (3) general expressions for the spin relaxation rates without specifying whether the scattering processes are elastic ($\hat{\mathcal{D}} = 0$) or inelastic ($\hat{\mathcal{D}} \neq 0$). To that end, we follow Ref. [4] and employ a perturbative approach with respect to the torque forces. Our results are therefore valid for $|\vec{\Omega}_{IA} + \vec{\Omega}_g|\tau_1 < 1$. Expanding $\vec{\Omega}_{IA}(\vec{k})$ and $\vec{\Omega}_g(\vec{k})$ in terms of spherical harmonics $Y_{lm}(\Theta, \Phi)$, we find ($i = 1, 2$) [15]

$$[T_i]^{-1} = \left[T_i^{EY}\right]^{-1} + \left[T_i^{DP}\right]^{-1} + \left[T_i^{VG}\right]^{-1}, \tag{4}$$

with the EY contributions (due to spin-flip scattering)

$$\left[T_1^{EY}\right]^{-1} = 2\left[T_2^{EY}\right]^{-1} = \frac{32\pi}{3} C_{sf}^2 \left\langle \frac{k^4}{\tau_1(k)} \right\rangle, \tag{5}$$

the DP contributions (due to inversion asymmetry)

$$\left[T_1^{DP}\right]^{-1} = 4|C_{31}^Y|^2\tilde{\tau}_{31}^3 + 4|C_{33}^Y|^2\tilde{\tau}_{33}^3, \tag{6}$$

$$\left[T_2^{DP}\right]^{-1} = 2|C_{32}^Z|^2\tilde{\tau}_{32}^3 + 2|C_{31}^Y|^2\tilde{\tau}_{31}^3 + 2|C_{33}^Y|^2\tilde{\tau}_{33}^3, \tag{7}$$

and the VG contributions (due to the wave vector dependence of the CB electron g-factor)

$$\left[T_1^{VG}\right]^{-1} = 4|D_{21}^Y|^2\tilde{\tau}_{21}^1, \tag{8}$$

$$\left[T_2^{VG}\right]^{-1} = |D_{00}^Z|^2\tilde{\tau}_{00}^1 + |D_{20}^Z|^2\tilde{\tau}_{20}^1 + 2|D_{21}^Y|^2\tilde{\tau}_{21}^3. \tag{9}$$

Here, C_{lm}^i and D_{lm}^i ($i = X, Y, Z$) are the expansion coefficients of $\Omega_{IA}^i(\vec{k})$ and $\Omega_g^i(\vec{k})$, respectively, and ($\nu = 1, 2, 3$)

$$\tilde{\tau}_{lm}^\nu = -\mathrm{Re}\langle C_l(k)\tau_{lm}^\nu(k)\rangle, \tag{10}$$

where the brackets denote an average over k defined as $\langle\ (\ ...\ )\ \rangle = \int_0^\infty dk k^2 f(k)\bar{f}(k)(...)/4\pi\int_0^\infty dk k^2 f(k)\bar{f}(k)$, with $\bar{f}(k) = 1 - f(k)$ and $f(k)$ the equilibrium Fermi distribution function. The generalized wave vector relaxation time $\tau_{lm}^\nu(k)$ satisfies a differential equation

$$[\hat{\mathcal{D}} + i(m\Omega_C + \Omega_L^\nu) - \frac{1}{\tau_l(k)}]\tau_{lm}^\nu(k) = C_l(k), \tag{11}$$

with $C_l(k) = C_{IA}k^3\delta_{l3} + C_g(B)k^2[\delta_{l2} + \delta_{l0}]$, $\Omega_L^\nu = \Omega_L[\delta_{\nu 1} - \delta_{\nu 2}]$, and $1/\tau_l(k) = l(l+1)/2\tau_1(k)$. The constants characterizing the three spin relaxation processes are, respectively, $C_{sf} = \delta^2(\Delta + 2\epsilon_g)R_0 m_0/2\Delta\epsilon_g m^*$, $C_{IA} = 2\delta_0/R_0 a_0^3$, and $C_g(B) = 2\mu_B B/R_0 a_0^2$, where $\delta^2 = 2\Delta^2/(\Delta + \epsilon_g)(2\Delta + 3\epsilon_g)$, Δ is the spin-orbit splitting, ϵ_g is the band gap, R_0 and a_0 are the Rydberg energy and the Bohr radius, respectively, m^* and m_0 are the CB electron mass and the mass of a bare electron, respectively, and μ_B is the Bohr magenton. The detailed form of $\hat{\mathcal{D}}$ depends on the scattering processes and does not concern us here. [15]

Note, as a consequence of the orthogonality of the angle dependences, the EY, DP, and VG spin relaxation rates are additive. The generalized relaxation rate $1/\tau_{lm}^\nu(k)$, on the other hand, is in general not proportional to the sum of the (on-shell) relaxation rates $1/\tau_l(k)$ because of inelasticity. A Matthiessen-type rule for $1/\tau_{lm}^\nu(k)$ only holds for elastic scattering (see below).

We are interested in the magnetic field dependence of the spin relaxation processes which, at least qualitatively, should not depend on the approximation adopted to describe the scattering events. In the following, we treat therefore all scattering processes in the elastic approximation and neglect $\hat{\mathcal{D}}$ in Eq. (11). Specifically, we take scattering on ionized impurities, acoustic phonons, and longitudinal optical (LO) phonons into account. The elastic approximation restricts our results to low enough temperatures, where electron-impurity scattering dominates, and to high enough temperatures, where electron-phonon scattering becomes essentially elastic.

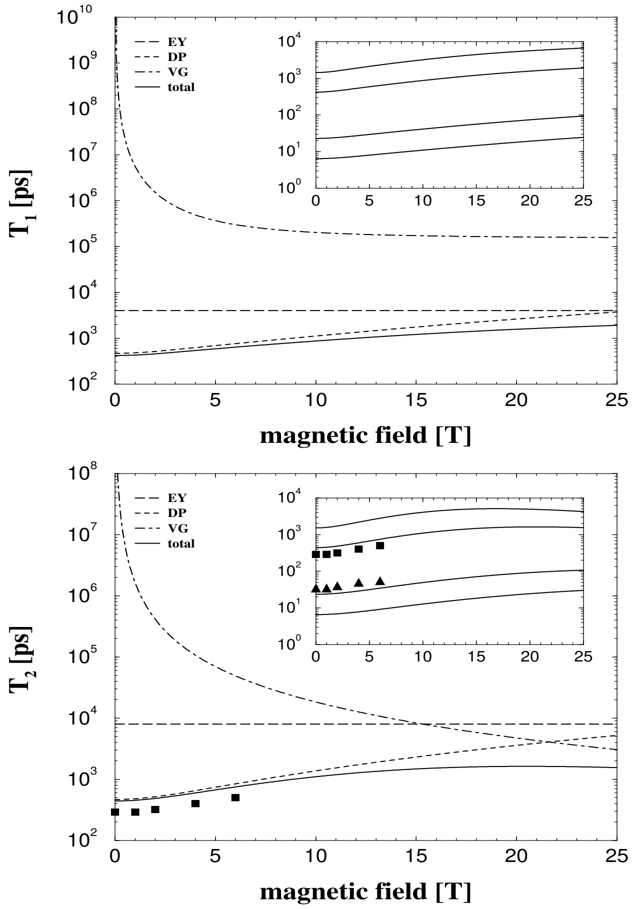

FIG. 1: The top and bottom panels show, respectively, T_1 and T_2 in GaAs as a function of magnetic field for $T = 0$ and $n = 10^{18}\mathrm{cm}^{-3}$. The contributions from the EY (long dash), DP (short dash) and VG (dot-dash) processes and the total relaxation time (solid) are shown in the main panel. The insets (same axis as the main panel) show the total relaxation times for $n = 5\times10^{17}\mathrm{cm}^{-3}, 1\times10^{18}\mathrm{cm}^{-3}, 5\times10^{18}\mathrm{cm}^{-3}$, and $1\times10^{19}\mathrm{cm}^{-3}$ (top to bottom). The squares and triangles are experimental data from Ref.[19] at the respective densities.

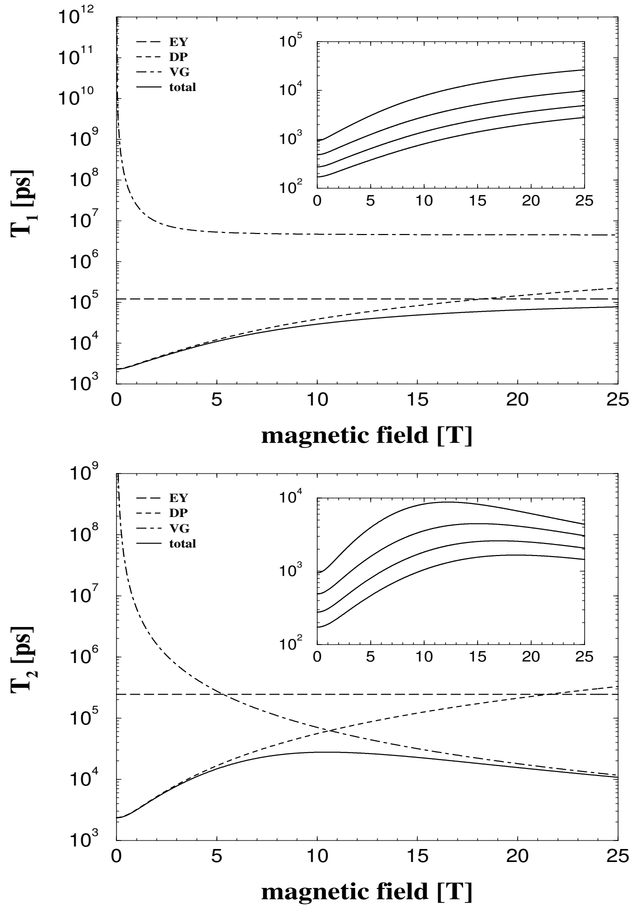

FIG. 2: The top and bottom panels show, respectively, T_1 and T_2 in GaAs as a function of magnetic field for $T = 100\mathrm{K}$ and $n = 10^{17}\mathrm{cm}^{-3}$. The contributions from the EY (long dash), DP (short dash) and VG (dot-dash) processes and the total relaxation time (solid) are shown in the main panel. The insets (same axis as the main panel) show the total relaxation times for $T = 150\mathrm{K}, 200\mathrm{K}, 250\mathrm{K}$, and $300\mathrm{K}$ (top to bottom).

Within the elastic approximation Eq. (11) reduces to an algebraic equation which is readily solved to yield

$$\tilde{\tau}^{\nu}_{lm} = \left\langle \frac{\tau_l(k)\left[C^2_{IA}k^6\delta_{l3} + C^2_g(B)k^4(\delta_{l2}+\delta_{l0})\right]}{1+[(m\Omega_C+\Omega^{\nu}_L)\tau_l(k)]^2} \right\rangle . \quad (12)$$

The k-average can be obtained either numerically or, at low and high temperatures, with saddle point techniques exploiting the peaked structure of the integrands. Within the elastic approximation it is sufficient to adopt the latter. Details of the calculation will be given elsewhere. [15]

In Fig. 1, we show calculated longitudinal (T_1) and transverse (T_2) spin relaxation times for GaAs as a function of magnetic field at $T = 0$ and an electron density of $n = 10^{18}\mathrm{cm}^{-3}$. We show separately the contributions to the spin relaxation times from the EY, DP and VG processes and the total spin relaxation time including all three spin relaxation processes. In the insets of Fig. 1, we give the total spin relaxation time for various electron densities at $T = 0$. The parameters needed to specify $\vec{\Omega}_g(\vec{k})$ have been previously obtained partly experimentally by measuring combined cyclotron resonances (a_4, a_6) and partly theoretically within a five-level Kane model (a_5): $(a_4, a_5, a_6) = (97, -8, 49) \times 10^{-24}\mathrm{eVcm}^2\mathrm{Oe}^{-1}$. [13] The parameter defining $\vec{\Omega}_{IA}(\vec{k})$ is given by $\delta_0 = 0.06\hbar^3/\sqrt{(2m^*)^3\epsilon_g}$. [16] The remaining parameters, such as the effective CB electron mass or the deformation potential are available from standard data bases. [17]

For the temperature and density conditions in Fig. 1, the electrons are degenerate and electron-ionized impurity scattering dominates. The VG process makes a small contribution to T_1 which is dominated by the DP process at zero magnetic field. As the magnetic field is increased, the DP process is quenched. Thus, T_1 increases monotonically with increasing magnetic field saturating at high field at a value determined by the EY process which is not affected by the magnetic field. [18] If the material parameters had been such that the EY process dominated the DP process for T_1 relaxation at zero magnetic field, T_1 relaxation would not be significantly affected by the applied field. By contrast the VG process makes a significant contribution to T_2 relaxation. At small applied magnetic fields the T_2 lifetime increases with increasing magnetic field, but as the field continues to increase the VG process begins to dominate the relaxation so that T_2 has a maximum and begins to decrease for larger magnetic fields. If the material parameters had been such that the EY process dominated the DP process for T_2 relaxation at zero magnetic field, the T_2 relaxation would monotonically decrease with increasing magnetic field.

The solid squares and triangles in the lower panel of Fig. 1 are measured T_2 spin lifetimes in GaAs at 5K from Ref. [19] at electron densities of $1 \times 10^{18}\mathrm{cm}^{-3}$ and $5 \times 10^{18}\mathrm{cm}^{-3}$. (Data for an electron concentration of $1 \times 10^{16}\mathrm{cm}^{-3}$ at 5K was also presented in Ref. [19], but at this low density the electrons are bound to isolated donors and our theory does not apply.) There is good (order of magnitude) agreement between our calculation and these measured results, although there were no adjustable parameters. Unfortunately, the magnetic fields in Ref. [19] are not high enough to capture any effects due to the VG process. In particular, our prediction of the maximum of T_2 remains to be experimentally verified.

In Fig. 2, we show the various contributions to the T_1 and T_2 spin relaxation for GaAs as a function of magnetic field at $T = 100$K and an electron density of $n = 10^{17}\mathrm{cm}^{-3}$. In the insets of Fig. 2, we show the total spin relaxation time as a function of magnetic field for various temperatures at $n = 10^{17}\mathrm{cm}^{-3}$. For the temperature and density conditions in Fig. 2, the electrons are non-degenerate and electron-LO-phonon scattering is the dominant scattering process. As for the degenerate electron case, the VG process makes a small contribution to T_1 which is again dominated by the DP process at zero magnetic field. The DP process is quenched by the field so that T_1 increases with field at small fields and saturates at a value determined by the EY process at large fields. [18] Similar to the degenerate electron case, the VG process makes a substantial contribution to T_2 relaxation. At small fields the T_2 lifetime increases with increasing field and at large fields the VG process begins to dominate the relaxation so that T_2 has again a maximum at some finite magnetic field. The sign of the slope in T_2 at small magnetic fields is again a clear signature of whether the EY process (T_2 decreases with increasing field) or DP process (T_2 increases with increasing field) dominates T_2 relaxation at zero magnetic field. Note, the qualitative behavior of the longitudinal and transverse spin relaxation times with increasing magnetic field is similar for degenerate and non-degenerate electrons, but the magnitude of the change is larger for non-degenerate electrons.

In summary, based on a systematic kinetic approach, which treats the EY, DP, and VG processes on an equal footing, we calculated the longitudinal (T_1) and transverse (T_2) spin relaxation times of CB electrons in n-type III-V semiconductors as a function of temperature, electron density, and magnetic field. At finite magnetic field, the VG process competes with the DP and EY processes. We find that, as a consequence of the interplay of the DP and the VG processes, T_2 can have a maximum as a function of magnetic field. In contrast, T_1 is not affected by the VG process and increases with magnetic field until it saturates at a value determined by the EY process. The sign of the change in T_2 with increasing magnetic field at small fields indicates, moreover, whether the EY process or the DP process dominates T_2 relaxation at zero magnetic field. Our calculated results are in good agreement with existing experimental data in n-type GaAs and we make additional specific predictions for the magnetic field dependence of electron spin lifetimes that are subject to experimental check.

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